**Barron’s Math 360: A Complete Study Guide to Geometry**

# Chapter 3: Angle Pairs and Perpendicular Lines

### What You Will Learn

* Complementary angles.
* Supplementary angles.
* Vertical angles.
* Definitions and theorems that intersect at angles.
* Alternative forms of a proof.

## Supplementary and Complementary Angle Pairs

### Definitions of Supplementary and Complementary Angles

* Two angles are *supplementary* if the sum of their measures is 180. .
* Two angles are *complementary* if the sum of their measures is 90. .

## Adjacent and Vertical Angle Pairs

Adjacent means “next to”. But how close do two angles have to be in order to considered adjacent?

|  |  |
| --- | --- |
|  |  |
| (a) Angles are adjacent | (b) Angles do not share the same vertex. |
|  |  |
| (c) Angles do not share a common side | (d) Angles overlap – they have interior points in common |

### Definition of Adjacent Angle Pairs

Two angles are an *adjacent pair* if they:

* Have the same vertex.
* Share a common side.
* Have no interior points in common.

If two angles are adjacent, the two sides that are not shared are sometimes referred to as *exterior* sides of the adjacent angles.

Theorem 3.1

If the exterior sides of a pair of adjacent angles form a straight line, then the angles are supplementary.

If two lines intersect, four angles are formed, as shown. Angles 1 and 3 are called vertical angles. Angles 1 and 4 are not vertical angles since they are adjacent. Notice that vertical angles are “opposite” one another. Angles 2 and 4 are also vertical angles.

A cross-section of a diagram

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### Definition of Vertical Angles

Vertical angles are pairs of non-adjacent (opposite) angles formed by two intersecting lines.

Name all the pairs of vertical angles in the accompanying diagram.

A black and white image of a triangle

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*Solution*: Angle pairs 1 and 4, 2 and 5, 3 and 6.

## Theorems Relating to Complementary, Supplementary and Vertical Angles

Angles A and B are each complementary to angle C.s

We may conclude that angle must be equal in measure (or congruent to) angle . By the same reasoning, if angles were each *supplementary* to angle , they would necessarily be congruent to each other.

Theorem 3.2

If two angles are complementary (or supplementary) to the same angle or to congruent angles, then they are *congruent*.

**This Is The Key To The Method!**

In approaching a proof, you need to plan thoughtfully and then organize the necessary steps in the “Statements” column. To maintain a logical train of thought, it is sometimes helpful to concentrate of completing the *entire* “Statements” column; then, to complete the proof, the corresponding reasons may be entered.

In the “Reasons” column, we will sometimes use the expressions “congruent” and “equal in measure” interchangeably.

Theorem 3.2

If two angles are complementary (or supplementary) to the same angle or to congruent angles, then they are *equal in measure*.

Theorem 3.3

Vertical angles are congruent.

## Definitions and Theorems Relating to Right Angles and Perpendiculars

Recall that a right angle is an angle of measure 90. The following theorems are useful in proving other theorems

Theorem 3.4

All right angles are congruent.

Theorem 3.5

If two angles are congruent and supplementary, then each is a right angle.

Two lines, two segments, or a line and a segment that intersect to form a right angle are said to be perpendicular. If line is perpendicular to line , we may write , when the symbol is read “is perpendicular to.”

A *perpendicular bisector* of a line segment is a line or segment that is perpendicular to the given segment at its midpoint.

A diagram of a graph

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### Definition of Perpendicular Lines

*Perpendicular lines* are lines that intersect to form right angles. If a line is perpendicular to a segment and intersects the segment at its midpoint, then the line is called the *perpendicular bisector* of the segment.

Theorem 3.6

If the exterior sides of a pair of adjacent angles are perpendicular, then the angles are complementary.

### Existence of Perpendiculars

How many perpendiculars can be drawn to a given line? A line has an infinite number of perpendiculars; however, it can be proven that through a particular point on the line there exists exactly one line (or segment) that passes through the point and is perpendicular to the line.

Theorem 3.7

Perpendicular lines intersect to form four right angles.

Theorem 3.8

Through a given point on a line, there exists exactly one perpendicular to the given line.

Postulate 3.1

Through a given point not on a line, there exists one perpendicular to the given line.

### Distance

The term *distance* in geometry is always interpreted as the *shortest* path between two points.

A point that is exactly the same distance from two other points is said to be *equidistant* from the two points.

The shortest path between two points is represented by the segments joining the points.

The shortest path from a point not on a line to the line measure by the length of the segment drawn from the point perpendicular to the line.

**Definitions of Distance**

* The *distance between two points* is the length of the segment joining the points.
* The *distance between a line and a point not on the line* is the length of the perpendicular segment drawn from the point to a line.

### Perpendicular Bisector

A *perpendicular bisector* of a segment is a line, ray or segment that is perpendicular to the given segment at its midpoint. Each point on the perpendicular bisector is equidistant from the endpoints of the segment.

### Methods for Proving Lines Are Perpendicular

By using the definition of perpendicularity, we may conclude that two lines are perpendicular if they intersect to form a right angle.

It will sometimes be convenient in subsequent work with perpendicular lines to use an alternative method.

If we continue to rotate a line in a clockwise fashion, eventually lines will be perpendicular, and this will be true when angles 1 and 2 are congruent.

Theorem 3.9

If two lines intersect to form congruent adjacent angles, then the lines are perpendicular.

A graph of a circle and a circle

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## A Word About the Format of a Proof

The two-column deductive geometric proof is a “formal” type of proof, since each statement is numbered and listed with a corresponding reason using a structured format. Sometimes it is convenient to use an “informal” proof in which the key steps of a proof are summarized in paragraph form. Informal proofs were given for Theorems 3.4 and 3.5.

Another type of proof that lends itself to a paragraph format is proof by counterexample. This type of proof is used to disprove a statement by providing a counterexample – a single, specific instance that contradicts a proposed generalization.

## Summary

* Two angles are congruent if they are:
  + Vertical angles formed by two intersecting lines.
  + Complements of the same or congruent angles.
  + Supplements of the same or congruent angles.
  + Right angles.
* If two angles are congruent and supplementary, then each is a right angle.
* To prove that two lines are perpendicular:
  + Show that the lines intersect to form a right angle.
  + Show that the lines intersect to form a pair of congruent adjacent angles (Theorem 3.7).

## Review Exercises

1. Two angles are complementary. The measure of one angle exceeds two times the measure of the other angle by 21. What is the degree measure of the smaller angle.

x – first angle

y – second angle

**(1) 23**

1. In the accompanying figure, what is the value of ?

**(2) 30**

1. In the accompanying figure, line is perpendicular to line . What is the value of ?

**(2) 20**

1. The measure of an angle is 5 times as great as the measure of its complement. Find the measure of the angle.
2. An angle measures four less than three times its supplement.

– angle

– supplement

1. The measure of the supplement of an angle is 3 times as great as the measure of the angle’s complement. Find the measure of the angle.

– angle

= angle’s supplement

1. The difference between the measures of an angle and its complement is 14. Find the measure of the angle and its complement.

– angle

– angle’s complement

1. Find the measure of an angle if it is 12 less than twice the measure of its complement.

– angle

– angle’s complement

1. Find the value of .

1. intersect at point .   
   If and , find .